THE MATRICULATION EXAMINATION BOARD | 22.03.2002 | MATHEMATICS BASIC COURSE

Ten problems at most may be answered. There are several items [marked **a)**, **b)** etc.] in some of the problems; all of them are to be considered for a complete answer.

1. **a)** Solve the equation $3x + 4 = 5 - 6x$. **b)** Solve the equation $12x^2 - 7x + 1 = 0$.

2. The area of a circle is 12 cm^2. What is the area of the square that circumscribes the circle? What is the area of the square inscribed in the circle? Give the answer in square centimeters with two decimals.

3. **a)** The budget of the Finnish state has been about 200 milliard (Amer. billion) marks for several years. If this money were distributed equally to all Finnish people, how much would everyone get (in marks)? The population of Finland is about five million.
b) A person is born in 1983. In which year will he or she be one gigasecond ($= 10^9$ seconds) old? Leap years are not to be considered.

4. How much greater, expressed as a percentage, is the sum of the first 999 terms of the arithmetical sequence

$$2,\ 4,\ 6,\ 8,\ \ldots$$

than the sum of its first 888 terms?

5. A new package contained 10% more rice crispies than earlier. At the same time the price was raised by 12%. As a consequence, 10% fewer packages were sold. Did the sales of rice crispies increase or decrease measured **a)** in the weight of rice crispies, **b)** in money? What was the percentage of the change?

6. A mobile phone mast is erected on a hill 32 metres above the surface of a nearby lake. The height of the mast is 120 metres. Find the angle (from the horizontal plane) where the red light at the top of the mast is seen by a person on the opposite side of the lake at a distance of 4.5 kilometres. Give the answer with the accuracy of 0.1 degrees.

7. Calculate $\left(x - \frac{1}{x}\right)^2 - \left(x - \frac{2}{x}\right)^2$, in which $x = 10,\ 100,\ 1000,\ 10000$. Give the answer as a decimal number with all decimals. Explain how the result is computed (the most important intermediate results from the calculator, possible simplification of the expression etc.)

8. A solution of salt and water contains 25 per cent salt. Diluted solutions are obtained by adding water. How much water must be added to one kilogram of the original solution in order to obtain a 10 per cent solution? Work out a graphic representation which gives the amount of water to be added in order to get a solution with 2–25 % of salt. The amount of water (in kilograms) to be added to one kilogram of the original solution must be on the horizontal axis; the salt content of the new solution as a percentage must be on the vertical axis.

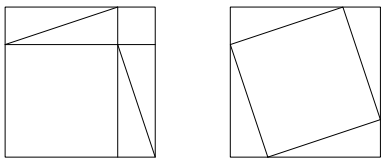
9. A die is tossed twice. Find the probability that in the second toss the face shows a greater number than in the first toss. If a die is tossed three times and the face in the first toss gives three, what is the probability that the second toss gives a higher face than the first and the third toss higher than the second?

10. The law of Archimedes says that a floating body has a mass equal to the mass of the water it displaces. A spherical hollow iron buoy is manufactured from iron plate and weighs 47 kg. Half of the buoy is below the surface. Find the thickness of the iron plate. The density of water is 1.0 kg/dm^3 and the density of iron is 7.7 kg/dm^3.

11. For which values of q is the polynomial function $f(x) = x^3 + x^2 + qx + 1$ decreasing on some interval? Find this interval.

12. A firm importing a new model of washing machines into Finland sold 470 machines in December 2000. An advertising agency suggested a marketing campaign that, starting at the beginning of 2001, would boost sales by 25% in each month, always compared with the preceding month, for two years. **a)** Find the number of washing machines the firm should sell in June 2001 according to the marketing proposal. **b)** Find the total number of machines to be sold during the two-year campaign.

13. Copy the accompanying figure onto your examination sheet. (Use a sufficiently large size.) Add necessary symbols to the figure and, with the aid of the figure, prove the Pythagorean theorem.



14. Two parents decided to open bank accounts for both of their children, Anne and Arthur, at the beginning of 2003 and to make a total deposit of 9 500 €. The aim was that the children would have equal sums at the beginning of the year following their 21st birthdays. What are the initial deposits in the accounts if Anne is 16 and Arthur 12 years old at the beginning of 2003? Suppose that the annual interest rate is 2.5%. Taxes are not taken into account.

15. A mathematics course had two consecutive tests; 242 students took part in both of them. Each test consisted of four questions and the maximum number of points was $4 \times 6 = 24$. In order to obtain some statistics, the following sums were calculated:

$$S_x = \sum x_i = 3805, \qquad S_y = \sum y_i = 1772,$$
$$T_x = \sum x_i^2 = 62741, \quad T_y = \sum y_i^2 = 18540, \quad U_{xy} = \sum x_i y_i = 29325,$$

where x_i and y_i refer to the number of points in the first and in the second test, respectively. Compute the mean value and the standard deviation of each test; find the correlation coefficient between the test results. Give the formulae needed for deriving the figures from the above sums.