



Questions 5, 6, 7, 9 and 10 have alternatives, of which only one can be chosen.

1. Find the solution  $x$  of the equation  $\frac{ax+b}{cx+d} = 3$ , when  
 $1^\circ a = 2, b = -3, c = 0, d = -5, \quad 2^\circ a = 1, b = 2, c = 4, d = 5.$
2. A student has obtained from the mathematics courses the grades 9, 8, 6, 4, 5 and 9. Find the mean and the median of these grades. The student takes one additional course in mathematics. How much can he or she improve the mean and the median with the grade of the new course?
3. The prenumeration price of a periodical is 558 mk and it must be paid not later than 15.4.1999. Alternatively, the price can be paid not later than 15.1.1999, in which case there is a discount of 15 mk. Find the annual rate of interest that causes both alternatives to be equally profitable. Which alternative is more profitable if the rate of interest is smaller?
4. The weight index is a common measure of fatness. The index is calculated by dividing the weight (in kilos) by the square of the height (in metres). A person has an ideal weight if the index of weight lies between 20 and 25. Find the number of kilos a person with a height of 183 cm and a weight of 91 kg has to lose in order to have an ideal weight. How many kilos does this person have to lose in order to be below the ideal weight?
5. a) Examine when the function  $f(x) = 7x^3 + x^2 - 40x + 36$  is increasing and when decreasing. Sketch the graph of the function.  
b) A hedge of uniform width and a height of 4 metres runs from east to west. The intention is to cut the hedge so that at noon there is sunshine on the shady side one metre nearer the hedge. At noon the sun is in the south at a height of 53 degrees. Find the amount the hedge has to be lowered by.
6. a) An electric company offers summer cottage owners two alternative deals for electricity, Home electricity 1 and Home electricity 2. The basic fee for the former is 62.50 mk/month and for the latter 57.50 mk/month. The prices for electricity are 35.6 p/kWh and 37.5 p/kWh, respectively. In both alternatives the electricity tax of 5.0935 p/kWh has to be paid, too. The value added tax of 22 % is included in the prices. Find the yearly consumption in which both deals are equally profitable. How large is the yearly bill then? Find the limits to the yearly consumption such that the annual bills of the deals differ less than 100 mk.  
b) One can draw 2.5 km gold wire from one gramme of gold. Find the diameter of such a wire with a circular cross section. Find the length of the gold wire if the diameter is 0.10 mm. The density of gold is  $19.3 \text{ kg/dm}^3$ .
7. a) Ball-shaped oranges of uniform size are packed in a cubical box: in a row one behind another, rows of equal length abreast, similar layers one on the other. The oranges of a layer are directly above the ones in the previous layer, not in the dimples. The size of the box is chosen so that the oranges are not able to move. Ball-shaped tangerines of uniform size are packed in the same way in an identical box. The diameter of a tangerine is a half of the diameter of an orange. Find in percentages how much larger the number of the tangerines in the box is than the number of oranges. Which box has a larger empty space? How much larger in percentages?

- b) Pillars are erected at 50 metre intervals on a 10 km stretch of a highway for illumination. The contractor fetches three pillars at a time from a depot by lorry, conveys the pillars to their places and returns to pick up the next lot. The depot is situated by the same highway two kilometres before the beginning of the stretch to be illuminated. The job starts from the depot and ends there. Find the minimum distance the contractor has to drive for the whole contract.
8. A vessel in the form of a circular cylinder stands under a rain pipe. The diameter of the cylinder is 52 cm and the height is 76 cm. It is known that a shower of rain of 1 mm brings 28 litres water to the vessel. In the region the monthly rainfall has a mean value of 42 mm and a standard deviation of 39 mm. Find the probability that the vessel will be full within one month when evaporation is disregarded. The amount of rain is normally distributed.
9. a) A factory accidentally polluted a pond of pure water so that the ratio of pollution was 20 ppm (ppm = a millionth part). Due to the weak current the water in the pond changes very slowly: only 2 % of its water is replaced by pure water annually. Find the number of years after which the water is again suitable for swimming, when the condition is that the ratio of pollution should be less than 2 ppm.
- b) Construct a function representing the distance of a point at the parabola  $y = x^2$  from the point  $P = (3, 1)$ ; choose the  $x$ -coordinate of a point at the parabola as variable. Find the point  $Q$  at the parabola which is nearest to the point  $P$  by calculating the smallest value of the distance — or, as well, the square of the distance. Use either bracketing or a graphical procedure for finding the zero of the derivative. Evaluate the coordinates of the point  $Q$  to the accuracy of one decimal place.
10. a) Both *arithmetic mean* and *geometric mean* are often called simply *mean* even if their definitions are different. The geometric mean of two positive numbers is the square root of their product. Examine by experimentation the reciprocal magnitude of the arithmetical and geometrical mean of two positive numbers  $x$  and  $y$ , especially their equality. Represent your result both verbally and using an inequality. Prove your inequality using, for instance, a square of a binomial.
- b) A school spends not more than 315 000 mk to buy computers for a classroom of 45 square meters. A desktop computer costs 11 500 mk and needs a space of 2.5 square meters. A laptop computer costs 17 000 mk but it can be used in a space of 1.5 square meters. How many computers of each type must be bought in order to have as many computers as possible in the classroom?