THE MATRICULATION EXAMINATION BOARD	21.9.2001	MATHEMATICS BASIC COURSE

Ten problems at most may be answered. There are several items [marked **a)**, **b)** etc.] in some of the problems; all of them are to be considered for a complete answer.

1. In some countries the Fahrenheit scale is used in measurements of temperature. A reading f on a Fahrenheit thermometer is transformed into the reading c on a Celsius thermometer by the formula $c = \frac{5}{9}(f - 32)$. How high a temperature does a person have expressed in degrees Fahrenheit, if the temperature is 38.2° in degrees Celsius? At which temperature does the reading on a Celsius thermometer coincide with the reading on a Fahrenheit thermometer?

2. A fresh birch log has a length of four metres and a diameter on average of half a metre. Find the mass of the log. The density of a fresh birch is roughly 0.9 $\mathrm{kg/dm^3}$.

3. The selling price of a product is obtained by adding to the taxfree price the value added tax, which is 22 per cent of the taxfree price. How much per cent is the value added tax of the selling price? Does this percentage depend on the selling price?

4. Solve the equation $4x^2 - 4ax - 3a^2 = 0$, when $a = 0.001$.

5. The distribution of the grades in a mathematics examination was the following:

4	5	6	7	8	9	10
1.3%	9.8%	15.8%	20.3%	23.3%	23.4%	6.1%

Find the mean of the grades.

6. The consumption amounts of a petrol-driven car and a diesel-oil-driven car are 7.9 litres and 5.4 litres per hundred kilometres, respectively. The price of one litre is 6.29 mk for petrol and 4.19 mk for diesel oil. For the diesel-oil-driven car one has to pay 2 700 mk as an annual diesel tax in contrast to the cheap diesel-oil price. Give the yearly expenses as a function of the yearly kilometres for both cars and draw the graphs of the functions in the same coordinate system, when the number of kilometres is below 30 000. Find the number of kilometres after which the diesel-oil-driven car will be cheaper to use.

7. The keeper of a kennel builds an enclosure for dogs that has five adjacent rectangle-shaped equal sections so that the whole enclosure forms a rectangle as well. He has material for a fence of exactly 200 metres. Find the dimensions of one section when the area of the whole enclosure is as large as possible. What is then the area of this enclosure?

8. Two cars are travelling, one behind the other, at a constant speed of 120 km/h. The distance between the cars is 150 metres. By the roadside a traffic sign shows the beginning of a speed limit zone of 100 km/h. Suppose that both drivers slow down immediately to 100 km/h when passing the traffic sign. Find the distance between the cars at the speed limit zone of 100 km/h. Find the expression for the distance in a general case where the initial speed is v_1, the slower speed v_2 and the initial distance d.

9. A faulty dice is cast in a computer game. The probability of a number is directly proportional to the number. Find the probabilities of different numbers. Find the probability to get twice 6 with two throws.

10. Suppose that a traffic tunnel is dug from Helsinki to Brussels so that it connects the cities rectilinearly along a chord of the globe. How deep does the deepest point of the tunnel lie? How many degrees has the gradient where it is driven into the tunnel at both ends? The radius of the earth is 6 370 km. The shortest distance between Helsinki and Brussels is 1 650 km as measured along the surface of the earth.

11. It is said that the famous mathematician Jacques Bernoulli deposited 58 Swiss francs in a bank in Basel at the beginning of the year 1699 and then forgot about the deposit. The bank paid an extremely high interest of 0.8 per cent on the deposit adding it to the capital at the end of the year. Find the year at the beginning of which the deposit had been doubled and the year it had respectively been quadrupled. Find the amount of the deposit at the beginning of this year.

12. On the musical scale c, d, e, f, g, a, b, c^1 there is a semitone between e and f and also between b and c^1; between all others there is a whole tone. In the *equal-tempered system* the semitones are of equal size, which means that the ratio of the number of oscillations of the notes f and e is the same as the one of c^1 and b; denote this ratio by k. In the whole tones the ratio of the number of oscillations of the note to the previous one is always the same and equal to k^2. Let the number of oscillations of c be 130. Then the number of oscillations of c^1 (one octave higher) is double, that is 260. Find the exact value of k as well as an approximate value. Evaluate the number of oscillations of the notes in the scale as whole numbers.

13. Compute $f(2) + f(3)$, when $f(x) = 1/x$. Show that $f(2) + f(3) \neq f(5)$. Further, show that there is no real number x ($x \neq 0$, $x \neq -2$) for which $f(2) + f(x) = f(2 + x)$.

14. A donation is given to the scholarship fund of an upper secondary school. What must be the amount of the donation in euros (€), if the donation and the interest are to be given as scholarships in the following fashion: 200 € exactly one year after the donation, 300 € two years after the donation, 400 € after three years, 500 € after four years and 600 € five years after the donation? The interest of 4.5% is added to the capital once in a year, the first time one year after the donation.

15. Alice, a student at the upper secondary school A, got 82 marks in a foreign language writing exercise common to all upper secondary schools. Bertha, a student from another upper secondary school B, got 80 marks. The mean in both schools was 72 marks. The standard deviation in school A was 9.2 marks and in school B 6.8 marks. Suppose that the number of marks is normally distributed. Investigate which of the students succeeded better compared with the level of her own school. How many students in school B, expressed as a percentage, did better than the mean value but worse than Bertha? How many students did better than Alice in school A?