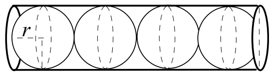
THE MATRICULATION EXAMINATION BOARD — 16.3.2001 — MATHEMATICS BASIC COURSE

Ten problems at most may be answered. There are several items [marked **a)**, **b)** etc.] in some of the problems; all of them are to be considered for a complete answer.

1. On a road-map some crossroads are marked with a circle with a diameter of 1.8 mm. What diameter of the crossing-area does this correspond to in reality, if the scale of the map is 1:200 000? If the diameter of the crossing-area is 25 m in reality, how large should it be on the map?

2. Solve the equation $x(x-\frac{7}{10})=\frac{1}{10}-x.$

3. A tightly packed container in the shape of a right-circular cylinder contains four tennis balls. What fraction of the volume of the container is the volume of the balls?



4. The Finnish Forest Research Institute measures the growth of the thickness of trees in certain stands of timber by using a sc. growth band, which registers the circumference of the tree at breast-height with one hundreth millimetre accuracy every hour. The measurements taken from a spruce exactly a year apart showed the following: In 1997 the circumference of the spruce was 102.20 cm. The measurement from 1998 showed that the circumference had increased by 13.16 mm, compared with the previous year; the measurement from 1999 showed 6.85 mm and the measurement from 2000 showed a 7.06 mm increase, always compared with the previous year. **a)** How much had the diameter of the spruce increased during this three-year period? Give the answer with 0.1 mm accuracy. **b)** By how much per cent had the cross-sectional area increased totally? The cross-section is assumed to be circular. Give the answer with one tenth of a per cent accuracy.

5. The turnover of a company was 11% smaller during the second quarter of a year than during the first quarter. The total turnover of the company was 6.0 million euro during that half year. How big was the turnover of the company during the first quarter of the year?

6. A straight line going through the points $(-2,11)$ and $(7,-1)$ forms a triangle together with the coordinate-axes. **a)** Form the equation of the line. **b)** Determine the lengths of the sides of the triangle formed.

7. The length of the leg AC in a right-angled triangle ABC is 8.6 cm and the length of the leg BC is 5.8 cm. On the leg AC there is a point D such that $DA = DB$. Find the lengths of the segments AB and CD.

8. A clinic for small animals uses an anaesthetic, which is administered in a single dose and the amount of which decreases exponentially in the body so that after three hours only half of it remains. During surgery there has to be at least 23 mg anaesthetic per kilo mass of the animal. How much anaesthetic at least has to be administered to a dog weighing 20 kg, if the surgery is expected to last for 1 h 15 min?

9. Cubes of decreasing size are stacked on each other. The edge of the first cube is exactly one metre and the edge of any other cube is half of that of the cube right underneath it. **a)** Determine the lengths of the edges of the three lowest cubes and of the n^{th} cube. **b)** What is the height of the stack consisting of the first 10 cubes? Calculate a table of the height of the stack, when it contains 11, 12, 13 and 14 cubes. If we imagine that the number of cubes grows without bounds, what number do the heights appear to approach?

10. Three of the ten letters in the word YLIOPPILAS are picked at random. What is the probability that **a)** the first letter picked is a vowel, **b)** only one of the picked letters is a vowel, **c)** the word ILO can be formed with the three letters picked?

11. An angle inscribed in an arc is an angle with its tip on the circumference of a circle and with its legs as chords of the circle. Let A and B be the endpoints of a diameter of the circle and let C be a third point on the circumference. Let O be the centre of the circle. Show that the angle inscribed in the arc ABC is half of the central angle AOC.

12. The amount of water in a tank that is being emptied is given in litres by $V(t) = 520(20-t)^2$, where t is time given in minutes. The velocity of the outflow is $q(t) = -V'(t)$ (litres/min). **a)** The emptying of the tank starts at time $t = 0$. When will the tank be empty? Sketch the graph of the function V. **b)** Determine the function q and sketch its graph. When is the velocity of the outflow at its largest?

13. The polynomials $P_n(x)$ named after the French mathematician Adrien Legendre are defined recursively as follows:

$$P_0(x) = 1, \quad P_1(x) = x,$$
$$P_n(x) = \frac{2n-1}{n}\, x\, P_{n-1}(x) - \frac{n-1}{n}\, P_{n-2}(x), \quad \text{when } n = 2, 3, 4, \ldots.$$

Determine the polynomials $P_2(x)$ and $P_3(x)$, corresponding to $n = 2$ and $n = 3$ and their derivatives $P_2'(x)$ and $P_3'(x)$. Show, that the equation $P_3'(x) - xP_2'(x) = 3P_2(x)$ holds.

14. Mrs. Virtanen, an engineer, had deposited 500 000 marks she got for selling an apartment in an account at the beginning of 1999, on which the bank paid an annual taxfree interest of 2.00 per cent. The taxation of deposit accounts was changed on June 1^{st}, 2000 so that from that day on all earned interest was taxed. A 29 per cent tax at source is deducted from the interest, and the bank collects it upon withdrawal, rounded off to whole marks. When the taxation was changed, the bank deposited in the account all the interest earned from the beginning of the year to that date. How much money did Virtanen get when she withdraw all the money from the account on August 25^{th}, 2000? The bank calculated the interest according to actual days from the day of deposit to the day of withdrawal, the day of withdrawal excluded. The bank calculated there were 366 interest days for the leap year 2000.

15. In the table are the points from the 100 metre race and the shot-put, and the final total points of the top ten decathletes at the Olympic Games in Sydney. Investigate to what extent the final result in the decathlon can be predicted from the combined result of the 100 metre race and the shot-put. Calculate for this purpose the coefficient of correlation between the sum of the points from these two events and the total points. Draw a correlation diagram of the distribution. Fit a line of regression to the diagram.

		100 m race	Shot-put	Total points
1.	Erki Nool EST	933	796	8641
2.	Roman Sebrle CZE	878	803	8606
3.	Chris Huffins USA	980	806	8595
4.	Dean Macey GBR	903	766	8567
5.	Tom Pappas USA	901	782	8425
6.	Tomas Dvorak CZE	881	846	8385
7.	Frank Busemann GER	881	760	8351
8.	Attila Zsivoczky HUN	838	787	8277
9.	Stefan Schmid GER	874	731	8206
10.	Henrik Dagård SWE	897	788	8178

Eduard Hämäläinen got 858 points in the 100 metre race and 732 points in the shot-put. What would Hämäläinen's expected final total points be if we used the results from the regression analysis to make a prediction?