THE MATRICULATION EXAMINATION BOARD | 26.3.1999 | MATHEMATICS ADVANCED COURSE

Questions 5, 6, 7, 9 and 10 have alternatives, of which only one can be chosen.

1. Solve the equations **1°** $(2x+1)(1-2x)(x+1)=0$, **2°** $(2x+1)(1-2x)(x+1)=1$.

2. In a company, research and development expenses were 12.0 % of the annual turnover in 1996. The following year the expenses were 2 % less of the annual turnover than in 1996. On the other hand, research and development expenses had increased 14 % in 1997. How much, expressed as a percentage, did the turnover of the company increase in 1997 compared with the previous year?

3. The sine of the angle between the base and a side in an isosceles triangle is $\frac{1}{3}$. Find the exact value and the approximate value, with three decimals, of the cosine of the vertex.

4. Find the derivative of $f(x)=e^{-\frac{1}{2}(x-\frac{3}{4})^2}$. Which values of x make $f'(x)$ negative?

5. **a)** Oil for a chain saw is sold in 1 litre and 3 litre plastic bottles. The bottles have the same shape and the amount of plastic is proportional to the surface area of the bottles. Find the ratio of the amount of plastic in the bottles. How much per cent less plastic has been used for each litre of oil in a 3 litre bottle than in a 1 litre bottle?

b) A community is expected to have 60 newborn children during the next year. The sexes of the children are independent and the probability that a child is a boy is 0.513. Which distribution is used to describe the number of girls and boys? Find the expectation values for the number of girls and for the number of boys. Find the probability that there are exactly the same number of girls and boys.

6. **a)** Let a, b and c be real numbers. Derive the formula for all real solutions of the equation $ax^2+bx+c=0$. (Note the case $a=0$ and the other special cases.)

b) During a revolution the piston of an automobile engine goes down and then up to the initial position. The wheel of an automobile turns at 780 revolutions per minute when the revolution counter of the automobile engine shows 4000 revolutions per minute. The diameter of the wheel is 73 cm. The distance from the highest position to the lowest position of the piston, i.e. the stroke, is 70 mm. Find the total distance traveled by the piston when the automobile is used for a trip of 60 kilometres.

7. **a)** A body is moving along the real axis and at the moment $t \geq 0$ it is at the point $x(t) = t^2 + |1-t|$. At what time is the body closest to the origin? Which points has the body passed in the time interval $[0,2]$?

b) Consider the vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ in the coordinate system with origin O. The points A, B and C form a triangle. One of the points is connected with a line segment to the midpoint of the opposing side and a point P divides this segment in the ratio $1:2$; the part meeting the opposing side is shorter. Find the formula for $\overrightarrow{OP}$ in terms of $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$. What conclusion can be made from this result?

8. A point (x,y) on the curve $y = \sin x$, $0 < x < \pi$, is connected to the x-axis with the straight line segment J parallel to the line $y = -\frac{1}{2}x$. Find the area A, expressed as a function of x, of the plane domain bounded by the curve in the interval $[0,x]$, the segment J and the x-axis. Find x such that A has the maximum value.

9. **a)** Consider the circles K_1: $x^2+y^2+4y=0$ and K_2: $x^2+y^2-10x-4y+28=0$ and the point $P=(4,0)$. Find the probability that a random line through P meets both K_1 and K_2.

b) A bacteria population $y(t)$ at time $t \geq 0$ satisfies the differential equation $y'(t) = ay(t) - by(t)^2$, where $a > b > 0$. Suppose that $y(t) \in]0, \frac{a}{b}[$ for each $t \geq 0$. Show, without solving the equation, that y is strictly increasing. Find the population $y(t_0)$ at $t_0 > 0$ provided that the rate $y'(t)$ of the growth of the population has the maximum value at t_0.

10. **a)** A function $f : \mathbb{R} \to \mathbb{R}$ is periodic, with a period $\omega \neq 0$, if $f(x+\omega) = f(x)$ for all $x \in \mathbb{R}$. Find an example of a continuous periodic function and of a discontinuous periodic function. Is the derivative of a periodic function periodic? Show that a continuous periodic function has a maximum value.

b) **1°** Show that the number

$$a_n 10^n + a_{n-1}10^{n-1} + a_{n-2}10^{n-2} + \cdots + a_1 10 + a_0$$

in the decimal number system is divisible by three if and only if $a_0 + a_1 + a_2 + \cdots + a_n$ is divisible by three. **2°** Show that the number $7^{2502} + 2^{1573}$ is divisible by three.