



Ten problems at most may be answered. There are several items [marked **a**), **b**) etc.] in some of the problems; all of them are to be considered for a complete answer.

1. Solve the linear system of equations

$$\begin{cases} 3x - 2y = 1, \\ 4x + 5y = 2. \end{cases}$$

2. Let $a \neq b$. Solve the equation $(a - b)x^2 + ax + b = 0$.
3. A train leaves Tampere at 8.06 and arrives in Helsinki at 9.58. A train in the opposite direction leaves Helsinki at 8.58 and arrives in Tampere at 11.02. The length of the journey is 187 kilometres. The trains are supposed to go at a constant speed and the time used for the stops is not counted. Find the average speed for both trains. At what distance from Helsinki and at what time do the trains meet?
4. Prove that the curves $x^2 - 6x - 3 + 4y = 0$ and $2x^2 + 1 - y = 0$ have one common point, where they are tangent to each other, i.e. they have a common tangent line. Find the equation of this line.
5. In a region the probability of rain is 60% if it has rained on the preceding day; the probability of dry weather is then 40%. If the weather has been dry on the preceding day, the probability of rain is only 20%, and the probability of dry weather 80%, respectively. Find the probability of rain for the day after tomorrow, when the weather is dry today.
6. The polar regions of the globe are the regions to the north of the Arctic Circle and to the south of the Antarctic Circle. The tropical region is on both sides of the equator between the northern and southern Tropics (Tropic of Cancer and of Capricorn). Find the area of the polar regions as a percentage of the area of the whole globe. Find also the percentage of the tropical region. The latitude of the polar (Arctic, Antarctic) circle is 66.5° and the latitude of the Tropic circle 23.5° .
7. Show that the line through the points $(2, 11\frac{1}{2}, 2)$ and $(4, \frac{1}{2}, -1)$ is perpendicular to the plane through the points $(5, 2, 0)$, $(1, 1, 1)$ and $(4, 1, 3)$.
8. Show, by considering the difference between y -coordinates, that the curve $y = \ln(1 + e^x)$ comes infinitely close to the line $y = x$ when $x \rightarrow \infty$.
9. Investigate for which values of $x \in \mathbb{R}$ the series

$$\sum_{k=0}^{\infty} \left(\frac{2x - 1}{3x + 1} \right)^k$$

converges. Find the sum of the series, as a function of x , and draw its graph.

- 10.** A part of a road has the form of a circular arc passing on a map through the points $(28, 98)$, $(70, 112)$ and $(126, 84)$ in an xy -coordinate system. Find the radius of the circle when the unit on the map corresponds to 25 metres in reality.
- 11.** Find the constant a such that the area of the region bounded by the lines $x = a$, $x = a + 2$, $y = 0$ and the curve $y = (x^2 + x + 1)^{-1}$ has the maximum value.
- 12.** Compute the volume for the solid of revolution generated by rotating the region bounded by the curve $y = x^3 + 1$, x -axis and the lines $x = 3$, $y = 9$ around the line $x = 3$.
- 13.** Let the functions f , g and h be defined as follows:

$$f(x) = \begin{cases} 0, & \text{if } x \neq 0, \\ 1, & \text{if } x = 0; \end{cases} \quad g(x) = \begin{cases} \sin(1/x), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0; \end{cases} \quad h(x) = xg(x).$$

- a) Find $\lim_{x \rightarrow 0} f(x)$.
- b) Find the zeros of the function g .
- c) Investigate which values the function g takes in the interval $[-0.01, 0.01]$.
- d) Find $\lim_{x \rightarrow 0} h(x)$.
- e) Is the function h continuous at the origin?
- f) Does the limit $\lim_{x \rightarrow 0} f(h(x))$ exist? Find it, if it exists.

Justify your answers briefly.

- 14.** Explain the Newton (Newton – Raphson) method for solving an equation numerically. Use the method for finding the greatest root of the equation $e^x + \sin x = 0$ with an accuracy of five decimals.
- 15.** In a society the growth of the standard of living is inversely proportional to the standard of living already gained, i.e. the higher the standard of living is, the less willingness there is to raise it further. Form a differential-equation-based model describing the standard of living and solve it. Does the standard of living rise forever? Is the rate of change increasing or decreasing? Does the standard of living approach some constant level?